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Study of Thermal Stresses in a Thin Annular Disc within the Context of Fractional Order Theory of Thermoelasticity

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ABSTRACT: In this paper, an attempt is made to determine the temperature distribution, displacement and stress functions on the outer curved surface of a time fractional two dimensional thermoelastic problem of thin annular disc occupying the space *D: a ≤r ≤b, -h ≤z ≤h* and of time fraction order derivative of order α. Further the inner and outer circular edges are kept at temperatures $u(z,t)$ and $g(z,t)$ and the corresponding lower and upper surfaces are kept at temperatures $F_1(r,t)$ and $F_2(r,t)$. The solution of the heat conduction problem is determined by applying the finite Marchi-Fasulo transform and Laplace transform techniques and the associated stresses are determined by using the displacement function.

Key words and phrases: Time Fractional, Fractional Calculus, Thin Annular Disc, Thermal Stresses, Integral Transforms Technique.

2000 Mathematics Subject Classification: 73.

I. INTRODUCTION

Fractional calculus deals with the analysis of studies that have several different possibilities of real or complex number powers of the differentiation operator D. Classical fractional derivatives include Grünwald-Letnikov derivative, Sonin–Letnikov derivative, Liouville derivative, Caputo derivative, Riesz derivative, Riesz–Miller derivative, Weyl derivative etc. And new fractional derivatives includes Machado derivative, Chen–Machado derivative, Caputo–Katugampola derivative, Caputo Fabrizio derivative etc. It is found that in many physical situations, the classical thermoelastic theory based on Fourier type heat conduction law breaks down like in case of manmade and biological materials/polymers, colloids, glassy and porous materials etc. So in above said cases, one needs to use a generalized theory of thermoelasticity which involve heat conduction model of time fractional (non-integer order) derivatives.

In [10, 11] Povstenko derived the variation of time-fractional differential operators with memory effects. Also time-fractional heat conduction in a composite medium is solved analytically for an infinite matrix and is presented for a spherical inclusion by Povstenko [12] and the corresponding associated thermal stresses. In [13-22], Kumar and Khobragade done mathematical modelling of some thermoelastic problems by the application of Fractional order theory.Raslan analyzed fractional order theory of thermoelasticity in a thick plate under axisymmetric temperature distribution and discussed its application [25]. The new thermoelastic fractional order theory was studied and developed by Sherief *et al.,* [26]. Xiong and Guo [27], derived the effect of variable properties with the action of moving heat source on magneto thermoelastic problem under application of fractional order thermoelasticity. Ezzat [28] did modeling of a new problem of the magneto-thermoelasticity theory within the context of a new consideration of fractional order derivative heat conduction. Ezzat [29] investigated the problem of state space approach to thermoelectric fluid with fractional order heat transfer. He uses Laplace transforms and state-space techniques to determine a one-dimensional application for a conducting half space of thermoelectric elastic material. In [1, 2] one dimensional transient thermoelastic problems derived the heating temperature and the heat flux on the surface of an isotropic infinite slab. The direct and inverse problems of thermoelasticity of a thin annular disc are considered in [3] to [8] Kumar and Khobragade [30] investigated three dimensional transient thermoelastic problem of a thin rectangular plate due to partially distributed heat supply. He determined temperature distribution, displacement and thermal stresses with the known boundary and initial conditions. Mirza and Roy [31] studied time-fractional three dimensional thermoelastic problem of a thin rectangular plate. Very recently Lamba and Deshmukh [23] studied hygrothermoelastic response of a finite solid circular cylinder.

In this present article, the mathematical model of fractional order thermoelastic problem for a thin annular disk with convective boundaries on the upper and lower surface by quasi-static approach is investigated. The temperature distribution, displacement and stress functions of a thin annular disc determined by applying finite Marchi-Fasulo transform and Laplace transform techniques.

II. STATEMENT OF THE PROBLEM

Consider a thin annular disc of thickness *2h* occupying the space *D:* $a \le r \le b,\ -h \le z \le h$. The differential equation governing the displacement function *U(r, z, t)* as [6] is

$$\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r}=(1+\nu)a_t T \tag{1}$$

with $(U_r)=0$ at $r = a$ and $r = b$ (2)

where ν and at are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the disc.
The Caputo type fractional derivative for nonlocal heat conduction is defined by [10] as

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha}=\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{d^n f(\tau)}{d\tau^n}d\tau, \qquad n-1<\alpha<n \tag{3}$$

To find Laplace transforms of the Caputo derivative it needs to know the initial values of the function $f(t)$ and its integer derivatives of the order $P=0,1,2,....,n-1$

$$L\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\}=s^\alpha f^*(s)-\sum_{P=0}^{P=n-1} f^{(P)}(0^+)s^{\alpha-1-P}, \qquad n-1<\alpha<n \tag{4}$$

The temperature distribution $T(r,z,t)$ of the plate is described by the differential equation of heat conduction in the context of fractional-order theory subjected to a time dependent heat flux in a thin annular disc as

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{k}\frac{\partial^\alpha T}{\partial t^\alpha} \tag{5}$$

subject to the initial condition

$$T(r,z,0)=0 \tag{6}$$

the boundary conditions

$$\left[T(r,z,t)+\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a}=u(z,t) \tag{7}$$

$$\left[T(r,z,t)\right]_{r=b}=g(z,t) \tag{8}$$

$$\left[T(r,z,t)-k_1\frac{\partial T(r,z,t)}{\partial z}\right]_{z=h}=F_1(r,t) \tag{9}$$

$$\left[T(r,z,t)+k_2\frac{\partial T(r,z,t)}{\partial z}\right]_{z=-h}=F_2(r,t) \tag{10}$$

where k_1 and k_2 are the radiation constants on the two plane surfaces, k is the thermal diffusivity of the material of the disc.
The stress functions σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr}=-2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{11}$$

$$\sigma_{\theta\theta}=-2\mu\frac{\partial^2 U}{\partial r^2} \tag{12}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz} and $\sigma_{\theta z}$ are zero within the disc in the plane state of stress. The Eqns. (1) to (12) constitute the mathematical formulation of the problem under consideration.

III. SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo integral transform and Laplace transform and their inversions to Eqn. (5) and making use of the transformed boundary and initial conditions (6)-(10), one obtains temperature distribution function expressed as follows,

$$\begin{aligned}T(r,z,t)=&\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\mu_m{}^2[Y_0(\mu_m a)+\mu_m Y_0'(\mu_m a)][Y_0(\mu_m\xi)+\mu_m Y_0'(\mu_m\xi)]}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0'(\mu_m a)]^2-[Y_0(\mu_m\xi)+\mu_m Y_0'(\mu_m\xi)]^2]}\\&\times[[Y_0(\mu_m a)+\mu_m Y_0'(\mu_m a)][J_0(\mu_m r)+\mu_m J_0'(\mu_m r)]-\\&[Y_0(\mu_m r)+\mu_m Y_0'(\mu_m r)][J_0(\mu_m a)+\mu_m J_0'(\mu_m a)]]\\&\times\bar{g}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right]\end{aligned}$$

$$-\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\mu_m{}^2[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2]}$$
$$\times[[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)][J_0(\mu_m r)+\mu_m J_0^{'}(\mu_m r)]-$$
$$[Y_0(\mu_m r)+\mu_m Y_0^{'}(\mu_m r)][J_0(\mu_m h)+\mu_m J_0^{'}(\mu_m h)]]$$
$$\times\bar{u}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right] \tag{13}$$

Where $L^{-1}\left[\frac{1}{s^\alpha+k(\mu_m{}^2+a_n{}^2)}\right]=E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)$

Here $E_\alpha(.)$ represents the Mittag-Leffler function.

where *m, n* are positive integer, μ_m are the positive roots of the transcendental equation

$$[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][J_0(\mu_m b)+\mu_m J_0^{'}(\mu_m b)]-$$
$$[Y_0(\mu_m b)+\mu_m Y_0^{'}(\mu_m b)][J_0(\mu_m a)+\mu_m J_0^{'}(\mu_m a)]]=0$$

$$\bar{g}(n,t)=\int_{-b}^{b}g(z,t)P_n(z)dz\ ,\ \bar{u}(n,t)=\int_{-b}^{b}u(z,t)P_n(z)dz\ ,\quad \lambda_n=\int_{-b}^{b}P_n^2(z)dz$$

$$P_n(z)=Q_n\cos(a_n z)-W_n\sin(a_n z)$$
$$Q_n=a_n(\alpha_1+\alpha_2)\cos(a_n h)+(\beta_1-\beta_2)\sin(a_n h)$$
$$W_n=(\beta_1+\beta_2)\cos(a_n h)+(\alpha_2-\alpha_1)a_n\sin(a_n h)$$

Eqn. (13) is the desired solutions of the given problem with $\beta_1=\beta_2=1$ and $\alpha_1=k_1$, $\alpha_2=k_2$.

IV. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting the value of *T(r, z, t)* from (13) in (1) one obtains the thermoelastic displacement function *U(r, z, t)* as

$$U(r,z,t)=-(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][Y_0(\mu_m\xi)+\mu_m Y_0^{'}(\mu_m\xi)]}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m\xi)+\mu_m Y_0^{'}(\mu_m\xi)]^2]}$$
$$\times[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][J_0(\mu_m r)+\mu_m J_0^{'}(\mu_m r)]-$$
$$[Y_0(\mu_m r)+\mu_m Y_0^{'}(\mu_m r)][J_0(\mu_m a)+\mu_m J_0^{'}(\mu_m a)]]$$
$$\times\bar{g}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right]$$
$$+(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2]}$$
$$\times[[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)][J_0(\mu_m r)+\mu_m J_0^{'}(\mu_m r)]-$$
$$[Y_0(\mu_m r)+\mu_m Y_0^{'}(\mu_m r)][J_0(\mu_m h)+\mu_m J_0^{'}(\mu_m h)]]$$
$$\times\bar{u}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right] \tag{14}$$

V. DETERMINATION OF STRESS FUNCTIONS

Using (14) in (11) and (12) the stress functions are obtained as

$$\sigma_{rr}=\frac{2\mu}{r}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\mu_m[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2]}$$
$$\times[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][J_0^{'}(\mu_m r)+\mu_m J_0^{''}(\mu_m r)]-$$
$$[Y_0^{'}(\mu_m r)+\mu_m Y_0^{''}(\mu_m r)][J_0(\mu_m a)+\mu_m J_0^{'}(\mu_m a)]]$$
$$\times\bar{g}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right]$$
$$-\frac{2\mu}{r}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m[Y_0(\lambda_m h)+\lambda_m Y_0^{'}(\lambda_m h)]^2}{(\lambda_m{}^2+a_n{}^2)[[Y_0(\lambda_m a)+\lambda_m Y_0^{'}(\lambda_m a)]^2-[Y_0(\lambda_m h)+\lambda_m Y_0^{'}(\lambda_m h)]^2]}$$
$$\times[[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)][J_0^{'}(\mu_m r)+\mu_m J_0^{''}(\mu_m r)]-$$
$$[Y_0^{'}(\mu_m r)+\mu_m Y_0^{''}(\mu_m r)][J_0(\mu_m h)+\mu_m J_0^{'}(\mu_m h)]]$$

$$\times\bar{u}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right] \tag{15}$$

$$\sigma_{\theta\theta}=2\mu(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\mu_m{}^2[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2]}$$

$$\times[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)][J_0^{''}(\mu_m r)-\mu_m J_1^{''}(\mu_m r)]-$$

$$[Y_0^{''}(\mu_m r)+\mu_m Y_1^{''}(\mu_m r)][J_0(\mu_m a)+\mu_m J_0^{'}(\mu_m a)]]$$

$$\times\bar{g}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right]$$

$$-2\mu(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\mu_m{}^2[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2}{(\mu_m{}^2+a_n{}^2)[[Y_0(\mu_m a)+\mu_m Y_0^{'}(\mu_m a)]^2-[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)]^2]}$$

$$\times[[Y_0(\mu_m h)+\mu_m Y_0^{'}(\mu_m h)][J_0^{''}(\mu_m r)-\mu_m J_1^{''}(\mu_m r)]-$$

$$[Y_0^{''}(\mu_m r)+\mu_m Y_1^{''}(\mu_m r)][J_0(\mu_m h)+\mu_m J_0^{'}(\mu_m h)]]$$

$$\times\bar{u}(n,t)\left[E_\alpha\left(-k(\mu_m{}^2+a_n{}^2)t^\alpha\right)\right] \tag{16}$$

Numerical Results and Discussion

Fixed, $u(z,t)=g(z,t)=\left(z^2-h^2\right)^2 e^{-at}$ where $a>0$

The constants associated with the numerical calculation are taken as:-
Inner radius a = 1 m,
Outer radius b = 2 m,
Thickness h = 0.1 m,

Material Properties: The cooper material was chosen for purpose of numerical calculation for a thin annular disc as:-

Thermal diffusivity k = 4.42 m^2/s

Density ρ = 558 kg/m^3

Specific heat c_p = 0.091 J/ (kg K)

Poisson ratio ν = 0.36

Coefficient of linear thermal expansion c_t = 16.5×10 −6 /K

Young's modulus E = 117 GPa

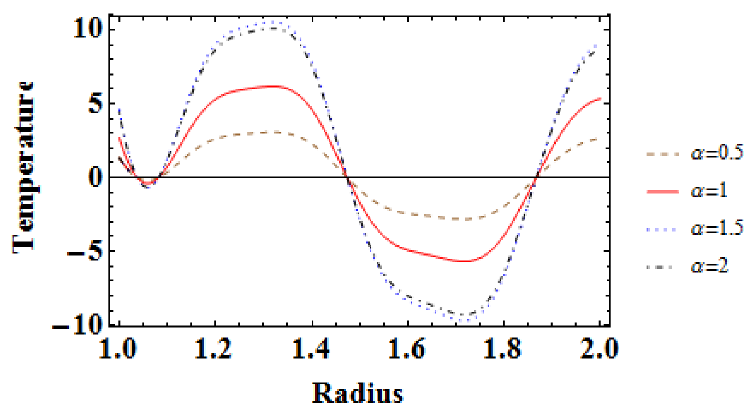

Fig. 1. Temperature distribution.

Fig. 1, represent the variation of temperature distribution along radial direction for the different values of fractional order parameters $a=0.5,\ 1,\ 1.5,\ 2$. It is seen that the nature of curve is sinusoidal. The temperature distribution flow non uniform pattern while moving from inner radii (r = 1) towards outer radii (r = 2). Also it is zero at the center r = 1.5 of the annular disk. Also it is noted that, the speed of propagation of the thermal signals is directly proportional to the values of fractional order parameter *a*.

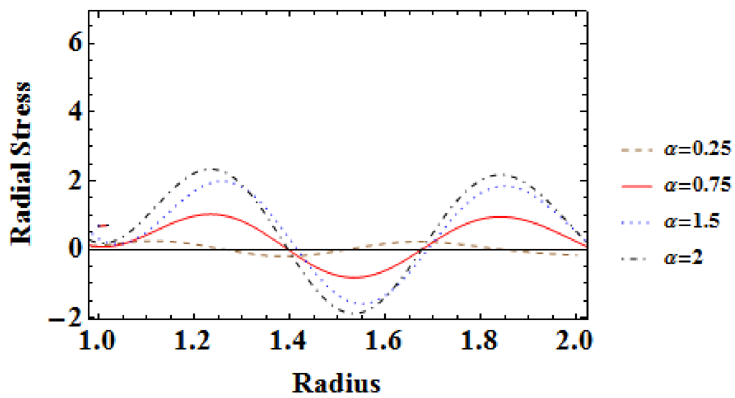

Fig. 2. Radial stress distribution.

Fig. 2 represent the variation of radial stress distribution along radial direction for the different values of fractional order parameters $a = 0.5, \ 1, \ 1.5, \ 2$. The radial stress is zero at both the inner and outer radii r = 1.and r = 2. It is seen that the nature of curve is sinusoidal. It is noted that, the stress propagation is directly proportional to the values of fractional order parameter a .

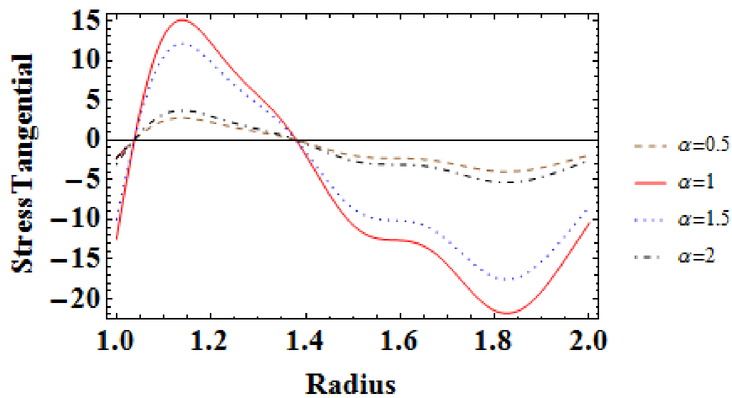

Fig. 3. Tangential stress distribution.

Fig. 3 shows the variation of tangential stress distribution along radial direction for the different values of fractional order parameters $a = 0.5, \ 1, \ 1.5, \ 2$. The tangential stress increases initially reaches to peak at r = 1.2 and then decreasing towards outer radii. Also it is seen observed that, the stress propagation is directly proportional to the values of fractional order parameter a .

Hence from the numerical results it is clear that time fractional order derivative significantly influence the temperature distribution as well as radial and tangential stresses. The weak and strong conductivity for the time fractional order parameter a noted within the range $0 < \alpha < 1$ and $1 < \alpha < 2$ and the normal conductivity represented by $\alpha = 1$.

VI. CONCLUSION

In this paper, we completely investigated the thermoelastic problem of a thin annular disc and determined the temperature distribution, displacement and thermal stresses on the outer curved surface of the disc with inhomogeneous third kind boundary condition based on fractional heat conduction with the Caputo time-fractional derivative of order $0 < \alpha \leq 2$. The finite Marchi-Fasulo transform and Laplace transform techniques have been used to find the solution of the problem. The results are obtained in terms of Bessel's function in the form of infinite series. A particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions. The discussed results in this paper will be very useful in studying the thermal characteristics of annular bodies in real-life science and engineering problems by considering the fractional derivative in the field equations.

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